



# Application of immune algorithm-based particle swarm optimization for optimized load distribution among cascade hydropower stations<sup>☆</sup>

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## ABSTRACT

The immune algorithm-based particle swarm optimization (IA-PSO), which is proposed by involving the immune information processing mechanism into the original particle swarm optimal algorithm, improves the ability to find the globally excellent result and the convergence speed with its special concentration selection mechanism and immune vaccination. Based on analyzing the model of load distribution among cascade hydropower stations and the traits of IA-PSO, the corresponding mathematical description and the solution procedure made with IA-PSO are given in detail. The result demonstrates that IA-PSO can achieve both a superior load distribution scheme and a higher convergence precision as compared to PSO, and will hopefully be applied to solving more extensive optimization problems.

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## 1. Introduction

Along with the employment of power mechanism innovations and advances in the general adoption of the market principle, every hydropower station or hydropower company will participate in market competition and price bidding as an independent electric power generator. Thereby, with the aim of making optimal use of a hydro power resource, it is very realistic and significant to study the load distribution among cascade hydroelectric power stations or multi-hydroelectric power stations belonging to the same owner.

Load distribution among cascade hydropower stations is different from in-plant economic operation of a single hydropower station, because it involves two issues: one is how to realize load distribution among hydropower stations, not involving the units; the other is how to distribute the load allocated in-plant for each station. Apparently it is a problem in multidimensional and nonlinear programming. Generally, traditional optimization methods, such as equal incremental equations [1], dynamic programming [2] and network flow [3], are unfeasible to find the optimum results because there are too many variables and constraints, which influences the calculation precision. In recent years, heuristic algorithms, such as the genetic algorithm [4] and the particle swarm optimal algorithm [5], which possess some superior traits: short-cut and general service, multiple calculation as well as low demand for constraint conditions, have attracted attention from electricity workers, and been applied to deal with load distribution, already obtaining some satisfying results. However, it is found in practice that as the algorithm advances, the individual multiplicity becomes uniform, resulting in these algorithms being liable to trapping in a partially extreme point [6,7], which is a bar to their extensive application to load distribution.

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Based on the PSO algorithm, the IA-PSO algorithm augmented by an immune system is proposed as a new evolution algorithm, which not only maintains the advantages of simplicity and easy operation, but overcomes the phenomena of premature convergence during the process of finding the optimized solution, as well as improving the ability of escaping from a partially extreme point. Therefore IA-PSO has a superior ability to find a globally excellent result and also has a higher convergence speed. In this paper, the corresponding IA-PSO model used to solve the problem of load distribution among cascade hydropower stations is proposed and the details of the solving process are given. Finally, through the case study and contrasting the results with those calculated by the PSO algorithm, the superiority of IA-PSO is verified.

## 2. The model of load distribution

In this paper, minimum water consumed is regarded as the optimizing principle to find the optimized power process of every available unit in a system on the basis of knowing the initial water level of every station and the system load curve. As a simplification, a cascade system made up of two stations is taken as an example. The upper station is the over annual regulation station, and the lower station is the daily regulating station. The time horizon of 24 h is divided into uniform time intervals and denoted by  $\varsigma = \{1, 2, \dots, T\}$ , let  $U_1, U_2$  denote the number of available units belonging to the upper and lower stations respectively. From this the set of indexes of the units,  $\varphi_i = \{1, 2, \dots, U_i\} i \in \{1, 2\}$  is constructed. The power flow of the upper and lower station is represented by the variable  $Q_{i,t}^1$  and  $Q_{j,t}^2 i \in \varphi_1, j \in \varphi_2, t \in \varsigma$  respectively. Let  $W_t^1, W_t^2, t \in \varsigma$  denote the start-stop water consumed by the unit switch respectively [8]. The variables  $q_{1,t}$  and  $q_{2,t}, t \in \varsigma$  are the waste water of the upper and the lower station respectively. The object function can be written as

$$F = \min \sum_{t=1}^T \left( \sum_{i=1}^{U_1} Q_{i,t}^1 + \sum_{j=1}^{U_2} Q_{j,t}^2 + W_t^1 + W_t^2 + q_{1,t} + q_{2,t} \right). \quad (2.1)$$

Subject to the following constraints:

a. Real power balance equilibrium

$$P_t = N_{1,t} + N_{2,t} \quad (2.2)$$

where  $P_t, t \in \varsigma$  denote the system load; the variables  $N_{1,t}$  and  $N_{2,t}, t \in \varsigma$  denote the power of the upper and lower station for each time-interval respectively.

b. Power constraints

$$N_{i,t,\min} \leq N_{i,t} \leq N_{i,t,\max} \quad (2.3)$$

where the variables  $N_{i,t,\min}$  and  $N_{i,t,\max} i \in \{1, 2\}$  denote the highest and lowest output constraint respectively.

c. Water balance equilibrium

$$\begin{cases} V_{1,t+1} = V_{1,t} + \eta(I_{1,t} - Q_{1,t}) \\ V_{2,t+1} = V_{2,t} + \eta(I_{2,t} - Q_{2,t}) \\ I_{2,t} = IQ_{2,t} + Q_{1,t-\tau} \end{cases} \quad (2.4)$$

where the variables  $V_{1,t}$  and  $V_{2,t}, t \in \varsigma$  denote the storage of the upper and lower stations respectively and the variables  $I_{1,t}$  and  $I_{2,t}, t \in \varsigma$  denote the average inflow for each time-interval; let  $\eta$  denote the unit conversion factor and  $IQ_{2,t}, t \in \varsigma$  denote the regional inflow into the lower station; the variables  $Q_{1,t}$  and  $Q_{2,t}, t \in \varsigma$  denote the discharge of the upper and lower stations respectively; let  $\tau$  denote the arrival time from the upper station to the lower one. Generally, it is set to be a constant to favor calculation.

d. Water level and storage capacity constraints

$$L_{i,t,\min} \leq L_{i,t} \leq L_{i,t,\max} \quad (2.5)$$

where the variables  $L_{i,t,\min}$  and  $L_{i,t,\max}, i \in \{1, 2\}, t \in \varsigma$  denote the highest and lowest water level constraints of stations. The lowest water level is the dead water level; the highest water level, determined according to the situation, is either the flood control water level in the flood season or the normal high water level in the non-flood season.

e. Other performance curve constraints of every station and unit, including the upper reservoir storage curve, discharge rating curve, water head characteristic, dynamic characteristics of the unit and the output constraints of the units etc.. More detailed information can be found in [9].

## 3. The principle of IA-PSO

### 3.1. The immune algorithm-based particle swarm optimal method (IA-PSO)

Actually, IA-PSO, which is proposed by involving the immune information processing mechanism into the original particle swarm optimal algorithm, is a kind of heuristic random algorithm with a stronger ability to find the globally excellent result. In an immune particle swarm system, the problem to be solved is regarded as an antigen, and every antibody represents one solution of the problem. At the same time, every antibody is also a particle of the particle swarm. The affinity between the antigen and the antibody, which is weighed by the fitness of particle, reflects the extent of satisfaction of the object function and the constraints. In addition, the affinity between antibody and antibody reflects the difference among

particles, namely the diversity of the particle swarm. However, during the renewal process of the particle swarm, it is preferable to preserve the particle with higher fitness. If this kind of particles becomes over concentrated, it is hard to keep the diversity of the swarm, which may plunge the algorithm into partially extreme optimization. So that, in contrast with PSO, on one hand, IA-PSO keeps the particles with different hierarchical fitness maintaining a certain concentration by using immune memory and a self-adjusting mechanism to guarantee the diversity of the particles; on the other hand, IA-PSO introduces immune vaccination, which can improve the ability of searching, and guide the process of evolution.

### 3.2. The realization process of IA-PSO algorithm

In order to realize the mathematical description of the objective function (2.1), let  $D$  denote the searching space and  $N$  denote the total amount in the particle swarm. Let  $\zeta = (1, 2, \dots, N)$  index the serial number of particles and denote the particle  $X_i \in D, i \in \zeta$ . Every particle in the space  $D$  can be described by a  $U \times T$  matrix. The variable  $U$  represents the number of all available units, and  $T$  is the number of regulation periods. Therefore, every element in a column vector denotes the power value of a certain unit at the corresponding period; and a row vector represents the power process of a certain unit during whole regulation stage. Let  $\varphi = (1, 2, \dots, U)$  and  $\phi = (1, 2, \dots, T)$  index the serial number of units and the serial number of time-intervals respectively, so that the power of a certain unit at a certain time-interval can be represented by the variable  $x_{u,t}, u \in \varphi, t \in \phi$ . The development of IA-PSO follows the steps:

(1) Initializing the particle swarm.

(2) Calculating a particle's fitness: the particle's fitness is a physical quantity indicating the magnitude of affinity between the antigen and the antibody. As the optimum result of the object function is the minimum water consumption of the cascade hydropower stations, the fitness function can be gained by transform from objective function.

$$f(X) = 1/F. \quad (3.1)$$

(3) Forming the next new generation particle swarm: The fitness function can be used to calculate a particle's fitness value, which indicates the virtue or defect degree of the particle's position. The particle adjusts its flight speed according to the gap between its current location and the two extremum registered. Let  $x_{g,d} \in D, x_{p,d} \in D$  denote the local best coordinate of the particle  $X_i \in D, i \in \zeta$  and the global best coordinate of the whole particle swarm respectively. Through flight, the next new generation particle swarm is formed. Let  $w \in R^+$  denote the inertia weight. Both  $C_1$  and  $C_2$  are acceleration coefficients.  $Rand() \in R^+$  is a random number, where  $Rand() \in (0, 1)$ . The maximum velocity is represented by the variable  $v_{i,\max} \in R_+, i \in \zeta$ , which is a constant set according to the concrete problem. Let  $v_{i,d} \in [-v_{i,\max}, v_{i,\max}]$  describe the velocity of the particle  $X_i \in D, i \in \zeta$  and  $x_{i,d}$  stands for the coordinate of the particle  $X_i \in D, i \in \zeta$ . Each particle has a velocity denoted by the variable  $v_{i,d}(k)$ , where  $k \in Z^+$  denotes the times of flight. So the calculation formulas for speed and position can be written as follows:

$$v_{i,d}(k+1) = w \times v_{i,d}(k) + C_1 \times Rand() \times (x_{p,d} - x_{i,d}) + C_2 \times Rand() \times (x_{g,d} - x_{i,d}) \quad (3.2)$$

$$x_{i,d}(k+1) = x_{i,d}(k) + v_{i,d}(k+1). \quad (3.3)$$

(4) Immune memory and self-adjustment: It is known from the evolution process of the particle swarm that every particle adjusts its "flying" direction to approach the best solution according to the search experience of whole particle swarm and itself. It is inevitable this will give rise to phenomena that certain kinds of particles, whose consistency belongs to same hierarchy, is excessive. In other words, the multiplicity of the swarm becomes uniform, and a new hyperplane cannot be provided during the evolution process of the swarm, thus the particle swarm reaches premature convergence because of trapping in a certain hyperplane, resulting in a lower search precision [10]. Therefore keeping diversity in the particle swarm is an important factor to avoid the algorithm trapping in a partial extremum. In the IA-PSO algorithm, the realization of the immune memory and self-adjustment mechanism provides an advantage in keeping diversity in the particle swarm.

The immune system will register the antibody, which reflects a certain ingredient of invading antigen, and when the invasion of the same kind of antigen occurs again, the memory cells will generate large numbers of corresponding antibodies. The process is called Immune Memory. In detail, in IA-PSO, the particle with the higher fitness is preserved as the memory particle used to substitute for the new generated particle, which does not meet the constraints (2.1)–(2.5). Self-adjustment is a kind of mechanism, which forces the system to generate the necessary antibodies in the right quantity, aiming at keeping the immunologic balance through suppressing or accelerating the antibodies, the consistency of which is high or low. In the IA-PSO algorithm, the self-adjustment mechanism is mainly aimed on keeping the particle, the sufficiency of which belongs to a different hierarchy, maintaining a certain concentration. The detailed steps are as follows.

–First, check whether all the new particles generated meet the constraints (2.1)–(2.5) or not, and substitute the memory particle for the unsatisfied ones.

–Second, let  $M \in Z^+$  denote the number of new particles generated on the basis of the new generation of the swarm, where  $M < N$ . Calculate the consistency of every particle, according to its fitness. Then, order all the particles from low to high. Choose the top  $N \in Z^+$  particles as the next new generation of the particle swarm.

The function to calculate the consistency of the particle  $X_i \in D, i \in \zeta$  is:

$$D(X_i) = \frac{1}{\sum_{j=1}^{N+M} |f(X_i) - f(X_j)|} \quad (j = 1, 2, \dots, N+M). \quad (3.4)$$

The selecting function on the basis of particle consistency probability can be written as

$$P(X_i) = \frac{\frac{1}{D(X_i)}}{\sum_{i=1}^{N+M} \frac{1}{D(X_i)}} \quad (j = 1, 2, \dots, N + M). \quad (3.5)$$

From the view of (3.5), the greater the number of particles similar to  $X_i$ , the lower the probability of the particles selected; on the contrary, the fewer the number of particles similar to  $X_i$ , the higher the probability of the particles not selected, thereby avoiding missing the particles with lower sufficiency while keeping a good evolution tendency.

(5) Immune vaccination: There are three main parts: picking-up vaccine, vaccination, and immune selection. Some characteristic information picked-up from a person's preknowledge about the problem to be solved, are regarded as bacterin used to change a certain integrant of the particle, aimed at guiding the search process. However, the postvaccinal particle must be checked by immune selection, which is capable of suppressing degradation phenomena. If the fitness of the postvaccinal particle is smaller than the original one, the original one will be preserved; otherwise, the postvaccinal particle will be regarded as the new particle and replace the original particle.

(6) Judging whether the process of evolution attains the terminal term: Generally, the terminal term is set as a maximum number of iterations or the fitness of the best particle meeting a preset threshold. On attaining the terminal term, the program will stop; otherwise, it returns to step (2), to continue the search.

#### 4. Application

For the application of IA-PSO to load distribution among cascade hydropower stations, the location of the particle in space stands for the power value of the unit, so the core idea of IA-PSO is to adjust the power value of the unit repeatedly to approach the optimum result gradually. There are some important steps, which should be concentrated on in solving the problem as follows.

1. Initialization of the particle swarm.
2. How to deal with the constraints.
3. Manufacture of the vaccine and the realization of immune vaccination.

1. Power balance equilibrium and water level and storage capacity equilibrium are two important constraints, which should be emphasized. Power balance equilibrium, which is a longitudinal balance, is mainly against the sum of unit power, which should be equal to the load allocated by the electric power system in any period within the regulation stage; water level and storage capacity equilibrium, which is a cross balance, depends on the power process of all units. If either of these bounds is broken, the particle initialized randomly is basically invalid. Because of the hydraulic relationship existing among cascade hydropower stations, the power and discharge of the upper station will influence the hydroelectric power condition of the lower station. In order to avoid calculating blindly, during the initialization of particle swarm, load distribution among cascade hydropower stations is carried out from the upper to the lower, with details as follows.

(a) According to the water head ratio at the beginning of the regulation stage, initialize the load distribution between the upper and lower station on the basis of meeting power balance and water level and storage capacity restrictions, and regard the load process distributed initially as an initial solution of this problem.

(b) On the basis of the initial solution, apply a stochastic disturbance to every time-interval load of the upper station, resulting in a new load distribution case. The function of the load disturbance is as follows:

$$N_{i,t} = N_{1,t}^0 + rand_2() \times (N_t - N_{1,t}^0) \quad (4.1)$$

$$N_{2,t} = N_t - N_{1,t} \quad (4.2)$$

$rand_2()$  is a random number giving a uniform distribution, where  $rand_2() \in [-1, 1]$ . The load of the upper station in the initial solution is represented by the variable  $N_{1,t}^0 \in R_+$ ,  $t \in \zeta$ .

(c) After working out the discharge process of the upper station, calculate the power process of the lower station, and check whether the operation process of the lower station meets the restrictions or not. If true, distribute the load randomly among  $U_i - 1$  ( $i = 1, 2$ ) units ahead within every station at each period, and the power process of the last unit is worked out in response to the power balance of each station; if not, return to the second step, applying a stochastic disturbance on the load process of the upper again, generating another new case of the load distribution.

2. In this paper the flight process of a particle is divided into two stages. In the first stage, find the serial number of the unit at each period, whose water consumption ratio is largest, and specify that the particle does not participate in flight. In the second stage, after the other units have adjusted their power values through flight, the time-interval power value of the unit not participating in flight can be worked out according to the power balance at each period. So that, on the condition of never reducing the capacity of finding the best value, the premise that every particle is a feasible solution of the problem during evolution process is guaranteed, improving the method of dealing with the bounds, which are broken in the flight process of the particle toward the two extremum. However, some characteristic bounds, such as gas erosion and vibration of unit, can be dealt with by using the penalty function method, which is described in [11,12] in more detail.

3. In an immune system, the vaccine, as a kind of estimation on a certain gene in the best antibody, is on the basis of some characteristic information from the preknowledge of the problem to be solved. However, with incomplete preknowledge,

**Table 1**

Datum of two stations.

Station	Regulation characteristic	Installed capacity (10 <sup>4</sup> kW)	Number of unit	Max passing flow (m <sup>3</sup> /s)	Water consumed by unit switch (m <sup>3</sup> /s)
A	Plurennial regulation	128	4	1190	45
B	Daily regulation	160	4	1450	44

**Table 2**

Power procedure of units.

Time-interval	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
System load	80	62	51	55	71	112	144	168	179	169	155	137	129	123	132	160	183	196	205	211	199	180	151	99
Unit power procedure of A station	1#	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	20	22	23	21	17	0	0
	2#	29	23	30	31	25	31	32	32	32	32	29	31	32	30	32	30	32	32	32	32	31	32	31
	3#	20	16	0	0	17	21	25	31	32	31	27	20	24	25	21	26	27	29	31	32	29	28	27
	4#	0	0	0	0	0	16	18	22	30	23	20	16	17	0	16	21	18	21	27	30	21	20	20
Unit power procedure of B station	1#	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	2#	0	0	0	0	0	18	31	25	27	26	21	19	23	29	28	24	29	30	30	30	32	27	33
	3#	31	23	21	24	29	26	38	39	39	39	36	35	34	37	37	39	40	40	40	40	39	39	26
	4#	0	0	0	0	0	0	19	19	18	19	18	0	0	0	18	23	24	23	24	24	18	0	0

it is hard to pick up any available characteristic information. From the view of the evolution process of particle swarm, after a certain number of generations, some certain gene segments of the best solution, namely some partial characteristic information of the particle, will have appeared in some antibodies, and can be picked up as the vaccine. So how we recognize these gene segments can be the key to picking a vaccine. The detailed process of realization is seen as follows.

Taking a unit selected randomly, for example, and dividing its power scope uniformly into  $m$  intervals. Let  $\xi = (0, 1, \dots, m)$  index the serial number of the power interval and denote these boundary power values  $N_l$ ,  $l \in \xi$ . The sign of the power section, where the variable  $x_{u,t}^i$ ,  $u \in \varphi$ ,  $t \in \phi$ ,  $i \in \xi$  is located can be defined as follows.

$$g(x_{u,t}^i) = \begin{cases} k_0 & x_{u,t}^i = 0 \\ k_1 & N_0 \leq x_{u,t}^i < N_1 \\ k_2 & N_1 \leq x_{u,t}^i < N_3 \\ \dots & \dots \\ k_m & N_{m-1} \leq x_{u,t}^i < N_m \end{cases} \quad (4.3)$$

where the variables  $N_{\min}$  and  $N_{\max}$  denote the minimum power restriction and the maximum power restriction of the unit respectively, where  $N_0 = N_{\min}$  and  $N_m = N_{\max}$ . The probability of the sign  $k_l$ ,  $l \in \xi$  at the place where the variable  $x_{u,t}$  is located can be calculated through the function  $p_l(x_{u,t}) = \frac{1}{N} \sum_{i=1}^N a_i$ , where  $a_i = \begin{cases} 1 & g(x_{u,t}^i) = k_l \\ 0 & \text{others} \end{cases}$ . At the beginning of initialization, particles are generated randomly so that it is hard to form stabilization at a certain partial variable  $x_{u,t}$ . However, along with a flight toward the two extremum, the probability of the sign  $k_l$  representing the partial variable  $x_{u,t}$  will increase. When the probability attains a preset threshold, the partial variable  $x_{u,t}$  can be regarded as the vaccine to be picked up. Then, select a particle randomly from the swarm, replace the value of the partial variance  $x_{u,t}$  at the corresponding place with the value of the vaccine, at the same time, in order to guarantee power balance, find the unit, the water consumption ratio of which is the largest, and replace its power value with the one calculated in response to the power value of the other units in the same period. All the above is the process of immune vaccination.

## 5. Case study

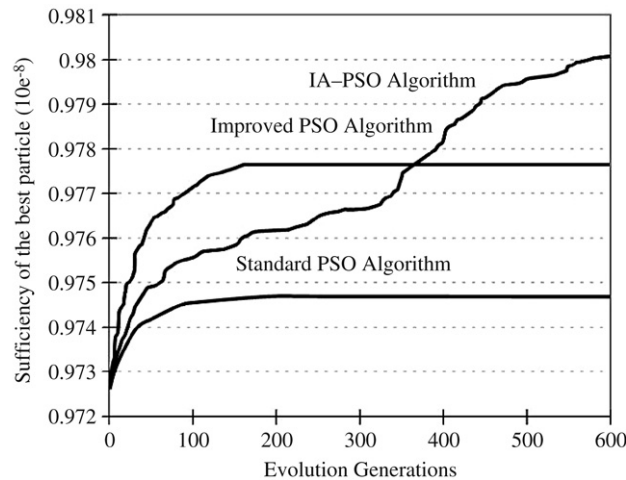
The case study concerns two stations, for details see Table 1. Here the load distribution between the two stations is solved with the IA-PSO algorithm compiled with the computer language C++. Final optimizing result is shown in Table 2. Comparing the result calculated with the standard PSO algorithm and the improved PSO algorithm introduced in Ref. [5] and the same initial condition, the superiority of IA-PSO algorithm is verified, for details see Table 3 and Fig. 1.

From the results above, it is known that the convergence of the IA-PSO algorithm is superior to the PSO algorithm. Additionally, although the convergence precision of IA-PSO is inferior to the one of the improved PSO proposed in Ref. [5] in the initial stage of evolution, the convergence precision of IA-PSO in the terminal stages is superior, which verifies the guidance effect of immune vaccine on evolution. Especially, at the terminal stage, the closer the value of the immune vaccine to the counterpart of the optimum solution, the better the effect of immune vaccination. Moreover, by considering the loss of water consumed by a start-stop of the unit in the objective function, the IA-PSO algorithm provides the best optimization

**Table 3**

Comparison of optimal results.

Optimizing algorithm	The number of unit switch within A station	The number of unit switch within B station	Water consumption ( $10^4 \text{ m}^3$ )
Standard PSO Algorithm	12	6	10258.692
Improved PSO algorithm	9	7	10227.811
IA-PSO algorithm	8	5	10203.794

**Fig. 1.** Comparison of the evolution process with three methods (standard PSO algorithm, improved PSO algorithm, IA-PSO algorithm).

result, namely reducing the number of start-stops of the units while minimizing water consumption, which is favorable for economic in-plant operation.

## 6. Conclusion

The issue of load distribution among cascade hydropower stations is a dynamic optimization problem with multiple dimensions and multiple stages. There are some deficiencies when solving this kind of problem in traditional ways. In this paper, the IA-PSO algorithm, which is proposed by involving the immune information processing mechanism into the PSO, is used to solve the problem, and the detailed steps are given. The algorithm possesses the traits of simple conformation and easy implementation. On one hand, adopting the selection mechanism based on particle concentration keeps particle swarm diversity against the phenomena of specificity; on the other hand, introducing the operation of immunization into the algorithm guides the search process and accelerates the speed of convergence. Through a case study and a comparison with the results from the other algorithms, a better load distribution was given by using the IA-PSO algorithm, so that IA-PSO algorithm could be regarded as a viable way to solve the problem of load distribution among cascade hydropower stations, and will hopefully be applied to solving more extensive optimization problems.

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